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Modified Yang–Mills Theory and Electroweak Interactions

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Since the Higgs boson of the standard electroweak model has not been detected despite many experimental attempts, nonstandard electroweak models not including the Higgs boson may be worthy of consideration; one of them is proposed here. This new model of electroweak interactions is based on the Yang–Mills theory completed by a nontrivial condition at infinity for the Yang–Mills potentials corresponding to the zero-field intensities. It is shown that within the framework of this model the three vector potentials of the Yang–Mills theory allow one to describe both the Maxwell electromagnetic interactions and the Fermi weak interactions and to obtain the known value of the Z^0 boson mass.

1. INTRODUCTION

Let us consider the Yang–Mills potentials $A^{k,\nu}$ satisfying the equations (Ryder, 1985)

$$\partial_\mu F^{k,\mu\nu} + g\varepsilon_{klm}F^{l,\mu\nu}A^m_\mu = (4\pi/c)J^{k,\nu} \tag{1}$$

$$F^{k,\mu\nu} = \partial^\mu A^{k,\nu} - \partial^\nu A^{k,\mu} - g\varepsilon_{klm}A^{l,\mu}A^{m,\nu} \tag{2}$$

where $\mu, \nu = 0, 1, 2, 3$; $k, l, m = 1, 2, 3$; $F^{k,\mu\nu}$ is the Yang–Mills tensor of field intensities, ε_{klm} is the antisymmetric tensor; $\varepsilon_{123} = 1$; g is the coupling constant; and $J^{k,\nu}$ are three 4-dimensional vectors of current densities.

As is well known, Eqs. (1)–(2) are covariant under the following infinitesimal transformations:

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$$J^{k,\nu} \to J^{k,\nu} + \varepsilon_{klm} J^{m,\nu}\theta^l, \qquad A^{k,\nu} \to A^{k,\nu} + \varepsilon_{klm} A^{m,\nu}\theta^l + (1/g)\partial^\nu\theta^k \tag{3}$$

where θ^l is a small angle of rotation of the three-dimensional vector ($J^{1,\nu}$, $J^{2,\nu}$, $J^{3,\nu}$) about the l axis.

For transformations (3) we also have (Ryder, 1985)

$$\begin{gathered} F^{k,\mu\nu} \to F^{k,\mu\nu} + \varepsilon_{klm} F^{m,\mu\nu}\theta^l \\ (J^{1,\nu})^2 + (J^{2,\nu})^2 + (J^{3,\nu})^2 \to (J^{1,\nu})^2 + (J^{2,\nu})^2 + (J^{3,\nu})^2 \end{gathered} \tag{4}$$

Consider spatial regions where there are no charged particles and hence $J^{k,\nu} = 0$. As follows from (3), in these regions solutions $A^{k,\nu}$ of Eqs. (1)–(2) depend upon three arbitrary functions θ^l of space-time coordinates. Therefore, in the regions where $J^{k,\nu} = 0$ we can complete Eqs. (1)–(2) by the Lorentz gauge for $A^{k,\nu}$:

$$\partial_\nu A^{k,\nu} = 0 \qquad \text{when} \quad J^{k,\nu} = 0 \tag{5}$$

In the spatial region occupied by charged particles we have the three equations of charge conservation,

$$\partial_\nu J^{k,\nu} = 0 \tag{6}$$

It must be noted that the system of equations (1), (2), (5), and (6) is covariant under the infinitesimal transformations (3) with $\theta^l = \text{const}$.

Let us add a condition at infinity for the potentials $A^{k,\nu}$ to Eqs. (1), (2), (5), and (6). For the Yang–Mills field intensities $F^{k,\mu\nu}$ we have

$$F^{k,\mu\nu} = 0, \qquad r \to \infty \tag{7}$$

where $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$ and x^1, x^2, x^3 are Cartesian spatial coordinates.

Let us find nonzero constant potentials $\overline{A}^{k,\nu}$ which satisfy (7) and can be limits of the potentials $A^{k,\nu}$ in a finite spatial region situated sufficiently far from the sources of the electroweak field. For these potentials we can write

$$A^{k,\nu} = \overline{A}^{k,\nu}, \qquad r \to \infty, \qquad \overline{A}^{k,\nu} = \text{const}, \qquad \sum_{k=1}^{3} \overline{A}^{k,\nu}\overline{A}^{k}_{\nu} \neq 0 \tag{8}$$

From (2), (7), and (8) we find

$$\varepsilon_{klm}\overline{A}^{l,\mu}\overline{A}^{m,\nu} = 0 \tag{9}$$

Let us assume that $\overline{A}^{1,0} \neq 0$. This can always be provided for the nontrivial potentials $\overline{A}^{k,\nu}$ under consideration by their gauge rotation and the choice of an inertial frame. Then for $\mu = 0$ and $k = 2, 3$ from (9) we get

$$\overline{A}^{1,0}\overline{A}^{3,\nu} - \overline{A}^{3,0}\overline{A}^{1,\nu} = 0, \qquad \overline{A}^{1,0}\overline{A}^{2,\nu} - \overline{A}^{2,0}\overline{A}^{1,\nu} = 0 \tag{10}$$

$$\text{i.e.,} \qquad \overline{A}^{3,\nu} = (\overline{A}^{3,0}/\overline{A}^{1,0})\overline{A}^{1,\nu}, \qquad \overline{A}^{2,\nu} = (\overline{A}^{2,0}/\overline{A}^{1,0})\overline{A}^{1,\nu}$$

Putting

$$\overline{\lambda}^k = \overline{A}^{k,0}/\overline{A}^{1,0}, \qquad \overline{\beta}^\nu = \overline{A}^{1,\nu} \tag{11}$$

from (10) and (11) we have the equality

$$\overline{A}^{k,\nu} = \overline{\lambda}^k \overline{\beta}^\nu \tag{12}$$

which is an obvious identity when $k = 1$.

It is easy to show that the constant potentials $\overline{A}^{k,\nu}$ having form (12) satisfy Eq. (9) for any indices k, μ, ν. This follows from the fact that the tensor ε_{klm} is antisymmetric and therefore $\varepsilon_{klm}\overline{\lambda}^l\overline{\lambda}^m = 0$. Since $\overline{A}^{k,\nu}$ are numbers satisfying Eq. (9), the potentials $A^{k,\nu} = \overline{A}^{k,\nu}$ satisfy Eq. (5) and condition (7) at infinity.

Putting

$$\lambda^k = \overline{\lambda}^k a, \qquad \beta^\nu = \overline{\beta}^\nu / a, \qquad a = (\overline{\beta}^\mu \overline{\beta}_\mu)^{1/2} \tag{13}$$

and using (8) and (12), we come to the following condition for the potentials $A^{k,\nu} = \overline{A}^{k,\nu}$ in a finite spatial region situated sufficiently far from the sources of the electroweak field:

$$A^{k,\nu} = \lambda^k \beta^\nu, \qquad r \to \infty, \qquad (\lambda^1)^2 + (\lambda^2)^2 + (\lambda^3)^2 = \Lambda^2, \qquad \beta^\nu \beta_\nu = 1 \tag{14}$$

where λ^k, β^ν, and Λ are some numbers. We will suppose that these numbers are real. It must be noted that condition (14) at infinity is covariant under Lorentz transformations of space-time coordinates.

As to the number Λ in (14), we regard it as a positive fundamental constant of the physical vacuum in spatial regions situated sufficiently far from the sources of the electroweak field. Then the condition $(\lambda^1)^2 + (\lambda^2)^2 + (\lambda^3)^2 = \Lambda^2$ in (14) is covariant under the infinitesimal transformations (3) with $\theta^l = \text{const}$ as well as the system of equations (1), (2), (5), and (6).

The system of equations (1), (2), (5), and (6) and condition (14) at infinity can be regarded as a modified Yang–Mills theory. As will be shown later, it can describe both electromagnetic and weak interactions and the constant Λ of this theory can be connected with the Fermi constant of weak interactions.

Let us first consider electromagnetic interactions. Then we put $J^{1,\nu} \neq 0$, $J^{2,\nu} = J^{3,\nu} = 0$ and find the following solution of Eqs. (1)–(2):

$$\begin{gathered} A^{2,\nu} = A^{3,\nu} = 0, \qquad F^{2,\mu\nu} = F^{3,\mu\nu} = 0 \\ F^{1,\mu\nu} = \partial^\mu A^{1,\nu} - \partial^\nu A^{1,\mu}, \qquad \partial_\mu F^{1,\mu\nu} = (4\pi/c) J^{1,\nu} \end{gathered} \tag{15}$$

which presents the classical Maxwell description of the electromagnetic field.

Therefore, the Maxwell equations are a particular case of the Yang–Mills equations

Consider now weak interactions in a small spatial region and represent potentials $A^{k,\nu}$ in the form

$$A^{k,\nu} = \lambda^k \beta^\nu + u^{k,\nu} \tag{16}$$

where, as follows from (14), $u^{k,\nu} \to 0$ when $r \to \infty$. For the considered weak interactions we assume that $|u^{k,\nu}| \ll \Lambda$. Then from (1), (2), and (16) we obtain the following linear equation for the small functions $u^{k,\nu}$ in which the small expressions of the second order are neglected:

$$\begin{aligned} &\partial^\mu \partial_\mu u^{k,\nu} - \partial^\nu \partial_\mu u^{k,\mu} - g\varepsilon_{klm}(2\lambda^l \beta^\mu \partial_\mu u^{m,\nu} + \lambda^m \beta^\nu \partial_\mu u^{l,\mu} + \lambda^m \beta_\mu \partial^\nu u^{l,\mu}) \\ &\quad - g^2 \lambda_m \beta_\mu (\lambda^k \beta^\nu u^{m,\mu} + \lambda^m \beta^\mu u^{k,\nu} - \lambda^k \beta^\mu u^{m,\nu} - \lambda^m \beta^\nu u^{k,\mu}) = (4\pi/c) J^{k,\nu}, \\ &\quad \lambda_m \equiv \lambda^m \end{aligned} \tag{17}$$

2. DESCRIPTION OF Z^0 BOSONS

Examine Eq. (17) for the Z^0 boson at rest. Then we have

$$J^{k,\nu} = 0, \qquad u^{k,\nu} = a^{k,\nu} \exp(-iM_{Z^0} c^2 t/\hbar) \tag{18}$$

where $a^{k,\nu} = \text{const}$, t is the time coordinate, and M_{Z^0} is the mass at rest of the Z^0 boson.

Since we consider a spherically symmetric problem and hence β^ν must not depend on the choice of space axes, we have to put the following, taking into account (14):

$$\beta^0 = \pm 1, \qquad \beta^1 = \beta^2 = \beta^3 = 0 \tag{19}$$

Puttting $\nu = 0$, from (18) and (5) we find

$$a^{k,0} = 0 \tag{20}$$

As follows from (18)–(20), Eq. (17) when $\nu = 0$ is an evident identity and when $\nu = 1, 2, 3$ we obtain from (17)–(20) the following linear equations for $a^{k,\nu}$:

$$\sum_{k=1}^{3} a^{k,\nu} d_{ik} = 0, \qquad \nu = 1, 2, 3, \quad i, k = 1, 2, 3 \tag{21}$$

where the matrix D with the elements d_{ik} has the form

$$D = (d_{ik})$$

$$= \begin{pmatrix} \omega^2 + g^2(\lambda_2{}^2 + \lambda_3{}^2) & 2ig\gamma\lambda_3 - g^2\lambda_1\lambda_2 & -2ig\gamma\lambda_2 - g^2\lambda_1\lambda_3 \\ -2ig\gamma\lambda_3 - g^2\lambda_1\lambda_2 & \omega^2 + g^2(\lambda_1{}^2 + \lambda_3{}^2) & 2ig\gamma\lambda_1 - g^2\lambda_2\lambda_3 \\ 2ig\gamma\lambda_2 - g^2\lambda_1\lambda_3 & -2ig\gamma\lambda_1 - g^2\lambda_2\lambda_3 & \omega^2 + g^2(\lambda_1{}^2 + \lambda_2{}^2) \end{pmatrix} \quad (22)$$

where

$$\lambda_k \equiv \lambda^k, \qquad \omega = M_{z^0}c/\hbar, \qquad \gamma = \beta^0\omega, \qquad \beta^0 = \pm 1 \qquad (23)$$

In order to have nonzero solutions to the three linear equations (21) we must put

$$\det(D) = 0 \qquad (24)$$

From (22)–(24), after calculating the determinant of the matrix D, we obtain the following equation for the mass M_{z^0}:

$$\det(D) = \omega^6 - 2g^2(\lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2)\omega^4 + g^4(\lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2)^2\omega^2 = 0 \qquad (25)$$

From (14), (23), and (25) we get

$$\omega^2(\omega^2 - g^2\Lambda^2)^2 = 0, \qquad \Lambda^2 = \lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2$$
$$\lambda_k \equiv \lambda^k, \qquad \omega = M_{z^0}c/\hbar \qquad (26)$$

and for the mass M_{z^0} of the Z^0 boson we find

$$M_{z^0} = g\Lambda\hbar/c \qquad (27)$$

3. WEAK FIELDS GENERATED BY NEUTRINOS

To connect later the constant Λ with the Fermi constant of weak interactions, let us examine electroweak fields described by Eq. (17).

Let us multiply Eq. (17) by λ^k. Then, since the tensor ε_{klm} is antisymmetric and hence $\varepsilon_{klm}\lambda^k\lambda^l = 0$, we easily get

$$\partial^\mu\partial_\mu y^\nu - \partial^\nu\partial_\mu y^\mu = (4\pi/c)\lambda_k J^{k,\nu}, \qquad y^\nu = \lambda_k u^{k,\nu}, \qquad \lambda_k \equiv \lambda^k \qquad (28)$$

Multiplying now Eq. (17) by β_ν, taking it again into account that the tensor ε_{klm} is antisymmetric, and using (14), we obtain

$$\partial^\mu\partial_\mu z^k - \beta_\nu\partial^\nu\partial_\mu u^{k,\mu} - g\varepsilon_{kim}(\lambda^l\beta^\mu\partial_\mu z^m + \lambda^m\partial_\mu u^{l,\mu})$$
$$= (4\pi/c)\beta_\nu J^{k,\nu} \qquad z^k = \beta_\nu u^{k,\nu} \qquad (29)$$

Consider the electron neutrino or muon neutrino. Then, since it is a neutral particle, we impose the following condition on the neutrino potentials:

$$u^{k,\nu} = 0 \qquad \text{outside the neutrino where } J^{k,\nu} = 0 \tag{30}$$

From (28) and (30) we find that in the region occupied by the neutrino

$$\lambda_k J^{k,\nu} = 0 \tag{31}$$

because if (31) had not been fulfilled we would get from (28) that outside the neutrino $\lambda_k u^{k,\nu} = y^\nu \neq 0$, contrary to (30).

From (28) and (31) we get

$$\lambda_k u^{k,\nu} = y^\nu = 0 \tag{32}$$

Since the mass of the neutrino is very small compared with the mass M_{z^0} of the Z^0 boson, we have the following correlation for the dimension r_ν of the region occupied by the neutrino, taking into account (27):

$$1/r_\nu \ll M/_{z^0}c/\hbar = g\Lambda \tag{33}$$

As follows from (33) and (14), we can neglect the linear terms with respect to λ^l on the left of Eq. (17) as compared with the quadratic terms. Therefore, from Eq. (17) we get the approximate equation

$$g^2\lambda_m\beta_\mu(\lambda^k\beta^\nu u^{m,\mu} + \lambda^m\beta^\mu u^{k,\nu} - \lambda^k\beta^\mu u^{m,\nu} - \lambda^m\beta^\nu u^{k,\mu}) = -(4\pi/c)J^{k,\nu} \tag{34}$$

From (32), (34), and (14) we have

$$u^{k,\nu} - \beta^\nu z^k = -4\pi J^{k,\nu}/cg^2\Lambda^2, \qquad z^k = \beta_\nu u^{k,\nu} \tag{35}$$

Using Eq. (6), from (35) we find

$$\partial_\nu u^{k,\nu} = \beta^\nu \partial_\nu z^k \tag{36}$$

Substituting (36) for $\partial_\mu u^{k,\mu}$ and $\partial_\mu u^{l,\mu}$ in (29) and taking it into account that the tensor ε_{klm} is antisymmetric, we obtain

$$\partial^\mu\partial_\mu z^k - \beta^\mu\beta^\nu\partial_\mu\partial_\nu z^k = (4\pi/c)\beta_\nu J^{k,\nu} \tag{37}$$

From (37) and (30) we find that in the region occupied by the neutrino

$$\beta_\nu J^{k,\nu} = 0 \tag{38}$$

because if (38) had not been fulfilled we would get from (37) that outside the neutrino $\beta_\nu u^{k,\nu} = z^k \neq 0$, contrary to (30).

From (37) and (38) we get

$$\beta_\nu u^{k,\nu} = z^k = 0 \tag{39}$$

and from (35) and (39) we find the weak field potentials as follows:

$$u^{k,\nu} = -4\pi J^{k,\nu}/cg^2\Lambda^2 \tag{40}$$

From (31) and (38) we have that the obtained potentials (40) satisfy correlations (32) and (39).

4. DETERMINATION OF CONSTANTS OF ELECTROWEAK INTERACTION

Let us introduce three electroweak charges q_k for particles taking part in electroweak interactions and represent their current densities $J^{k,\nu}$ in the form

$$J^{k,\nu}/c = q_k I^\nu \tag{41}$$

where I^ν is a 4-dimensional vector proportional to the particle mass density and the 4-dimensional vector of the particle velocity. We will assume that for the particles generating the electrical field, such as the proton and electron, the charge q_1 coincides with the electrical charge and $q_2 = q_3 = 0$. The charges of neutrinos will be considered later.

Let us seek a correlation for the charges of a charged elementary particle. This correlation must satisfy the following two properties:

1. The correlation must be covariant under transformations (3) preserving the Yang–Mills equations.
2. The charges $q_1 = \pm e_p$, $q_2 = q_3 = 0$ corresponding to the proton and electron, where e_p is the proton charge, must satisfy the correlation.

Taking into account (4), we find the following equality that fulfills the two properties:

$$q_1{}^2 + q_2{}^2 + q_3{}^2 = e_p{}^2 \tag{42}$$

Consider neutrinos. Since they are neutral particles having weak interactions with the electron, we assume that they have two charges q_1, q_2 or q_1, q_3 neutralizing each other, namely

$$q_1 = -q_2, \quad q_3 = 0 \qquad \text{or} \qquad q_1 = -q_3, \quad q_2 = 0 \tag{43}$$

Then from (42) and (43) we get

$$q_1 = \pm 2^{-1/2} e_p, \quad q_2 = -q_1, \quad q_3 = 0 \qquad \text{or} \qquad q_2 = 0, \quad q_3 = -q_1 \tag{44}$$

We assign the first case of (44), $q_2 = -q_1$, to the electron neutrino, and the second case, $q_3 = -q_1$, to the muon neutrino.

Let us turn now to the Dirac equation for a fermion with the charges q_1, q_2, q_3. It can be represented in the generalized form

$$i\hbar\, \partial\Psi/\partial t = [c\alpha_l(i\hbar\, \partial^l - q_k A^{k,l}/c) + \gamma^0 mc^2 + q_k A^{k,0}]\Psi,$$
$$\alpha_l = \gamma^0\gamma^l, \quad l = 1, 2, 3 \tag{45}$$

where Ψ is four wave functions of the fermion, γ^n are the Dirac matrices, m is the fermion mass at rest, and $A^{k,\nu}$ are electroweak field potentials.

Consider the gauge transformations (3) not changing the fermion current densities $J^{k,\nu}$. Then from (3) and (41) in the region occupied by the fermion we have

$$\theta^l = \eta q_l, \qquad J^{k,\nu} \to J^{k,\nu},$$
$$A^{k,\nu} \to A^{k,\nu} + \varepsilon_{klm}\eta q^l A^{m,\nu} + (1/g)q^k\partial^\nu\eta, \qquad q^l \equiv q_l \tag{46}$$

Taking into account that $\varepsilon_{klm}q^k q^l = 0$, from (45) and (46) we have the following gauge transformation not changing the fermion state:

$$q_k A^{k,\nu} \to q_k A^{k,\nu} + (1/g)q_k\partial^\nu\theta^k,$$
$$\Psi \to \Psi \exp(-iq_k\theta^k/\hbar cg), \qquad \theta^k = \eta q_k \tag{47}$$

It must be noted that in Rabinowitch (1996) a generalization for nucleons of the Dirac equation was proposed which describes their quark structure and anomalous magnetic moments. Consequently, for better description of the proton and neutron the Dirac equation should be replaced by this generalization of it.

The potentials $A^{k,\nu}$ in (45) are represented by formula (16). As to their constant part $\lambda^k\beta^\nu$, it can be removed in (45) by the gauge transformation $\Psi \to \Psi \exp(iq_k\lambda^k\beta_\nu x^\nu/c\hbar)$, where x^ν are the space-time coordinates in (45). Therefore, from (16) and (45) we find that the Hamiltonian of interaction H_{int} has the form (Bjorken and Drell, 1964)

$$H_{\text{int}} = q_k I_\nu u^{k,\nu}, \qquad I^\nu = \overline{\Psi}\gamma^\nu\Psi, \qquad \overline{\Psi} = \Psi^+\gamma^0 \tag{48}$$

where $J^{k,\nu} = cq_k I^\nu$ are current densities and Ψ^+ is the Hermitian conjugate of Ψ.

For the electron charges we have $q_1 = -e_p$, $q_2 = q_3 = 0$. Consider the electron in the weak field of its antineutrino. Since in this case the energy of interaction is positive, from (44), (48), (40), and (41) we get that for the antineutrino $q_1 = -q_2 = 2^{-1/2}\, e_p$, $q_3 = 0$.

Therefore, from (40), (41), and (48) we find that the Hamiltonian of interaction of the electron and antineutrino has the form

$$H_{\text{int}} = 2^{-1/2}4\pi\, e_p^2\, I^\mu_{(e)} I_{\mu(\tilde{\nu})}/g^2\Lambda^2 \tag{49}$$

where the indices (e) and $(\tilde{\nu})$ correspond to the electron and its antineutrino, respectively.

In the considered case the Hamiltonian of interaction can be represented in the form (Ryder, 1985)

$$H_{\text{int}} = 2^{-1/2} G_F I^{\mu}_{(e)} I_{\mu(\tilde{\nu})}, \qquad G_F = 10^{-5}\hbar^3/cm_p^2 \tag{50}$$

where G_F is the Fermi constant and m_p is the mass at rest of the proton.

Therefore, comparing (49) and (50), we get the equality

$$G_F = 10^{-5}\hbar^3/cm_p^2 = 4\pi(e_p/g\Lambda)^2 \tag{51}$$

From (51) and (27) we find

$$10^{-5}\hbar^3/cm_p^2 = 4\pi(e_p\hbar/M_{z^0}c)^2 \tag{52}$$

Equality (52) gives the following value of the mass at rest M_{z^0} of the Z^0 boson:

$$M_{z^0} = 200 m_p (10\pi\, e_p^2/\hbar c)^{1/2} = 89.8 \text{ GeV} \tag{53}$$

As is seen from (53), the obtained theoretical value of the mass M_{z^0} of the Z^0 boson accords with its experimental value (Ryder, 1995).

As is well known, the mass M_W of $W^{\pm}$ bosons is approximately equal to 80 GeV. The difference between the masses of Z^0 and $W^{\pm}$ bosons can be explained by the availability of the electric field energy of the latter.

Consider now the connection between the constants g and e_p. For this purpose let us examine the electron described by the Dirac equation (45) when the electroweak potentials $A^{k,\nu}$ are subject to the gauge transformation (46).

Since for the electron $q_1 = -e_p$, $q_2 = q_3 = 0$, from (46) we get

$$\begin{gathered} A^{1,\nu} \to A^{1,\nu} + (1/g)\partial^{\nu}\theta^1, \qquad A^{2,\nu} \to A^{2,\nu} - A^{3,\nu}\theta^1, \\ A^{3,\nu} \to A^{3,\nu} + A^{2,\nu}\theta^1 \end{gathered} \tag{54}$$

where θ^1 is a small angle of the rotation of the two-dimensional vector ($A^{2,\nu}$, $A^{3,\nu}$) about its origin.

Making such rotation N times by the small angle θ^1/N, where N is a large number, we easily generalize transformation (54) for an arbitrary angle θ^1 of the rotation of the two-dimensional vector ($A^{2,\nu}$, $A^{3,\nu}$) about its origin as follows:

$$\begin{aligned} &A^{1,\nu} \to A^{1,\nu} + (1/g)\partial^{\nu}\theta^1 \\ &A^{2,\nu} \to A^{2,\nu}\cos\theta^1 - A^{3,\nu}\sin\theta^1 \\ &A^{3,\nu} \to A^{3,\nu}\cos\theta^1 + A^{2,\nu}\sin\theta^1 \end{aligned} \tag{55}$$

For the electron wave function Ψ from (47) we get the gauge transformation

$$\Psi_{\theta^1} = \Psi_0 \exp(ie_p\theta^1/\hbar cg) \tag{56}$$

where Ψ_{θ^1} is the wave function Ψ corresponding to the angle θ^1 in (55).

Let us assume that in the spatial region occupied by an electron at rest the angle θ^1 slowly changes from the constant value $\theta^1 = 0$ to the constant value $\theta^1 = 2\pi$. Then, as follows from (55), at two moments when $\theta^1 = 0$ and when $\theta^1 = 2\pi$, the potentials $A^{k,\nu}$ are the same. Since these potentials determine the wave functions of the electron under consideration, we come to the conclusion that at the two moments when $\theta^1 = 0$ and $\theta^1 = 2\pi$ the wave functions Ψ_0 and $\Psi_{2\pi}$ are also the same. Hence, taking into account (56), we find

$$\Psi_{2\pi} = \Psi_0 \exp(2\pi ie_p/\hbar cg) = \Psi_0 \tag{57}$$

From (57) we easily get the following formula for the elementary charge e_p:

$$e_p = \hbar cg \tag{58}$$

Formula (58) explains the existence of the elementary charge e_p since in it this charge is determined by the fundamental constants $\hbar$, c, and g and gives the value $e_p/\hbar c$ of the constant g.

From (51) and (58) we find the value of the constant Λ:

$$\Lambda = 2\hbar c(\pi/G_F)^{1/2} = 1121m_p(c^3/\hbar)^{1/2} = 2.997 \times 10^8 \text{ g}^{1/2} \text{ cm}^{1/2}/\text{sec} \tag{59}$$

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